Taming Large MDPs Through Stochastic Games

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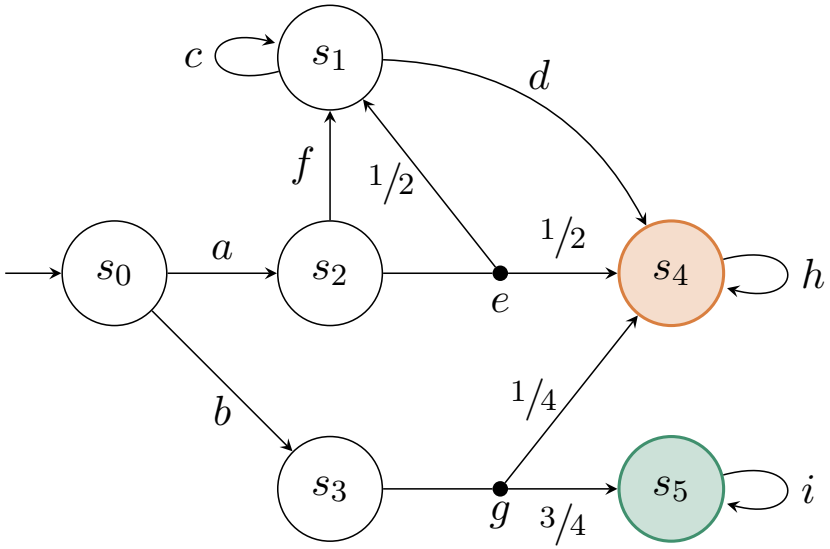
September 19, 2024

Highlights of Logic, Games and Automata 2024

Context



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- A finite set of actions A;
- A probabilistic transition function $\delta : S \times A \to \text{Dist}(S)$.

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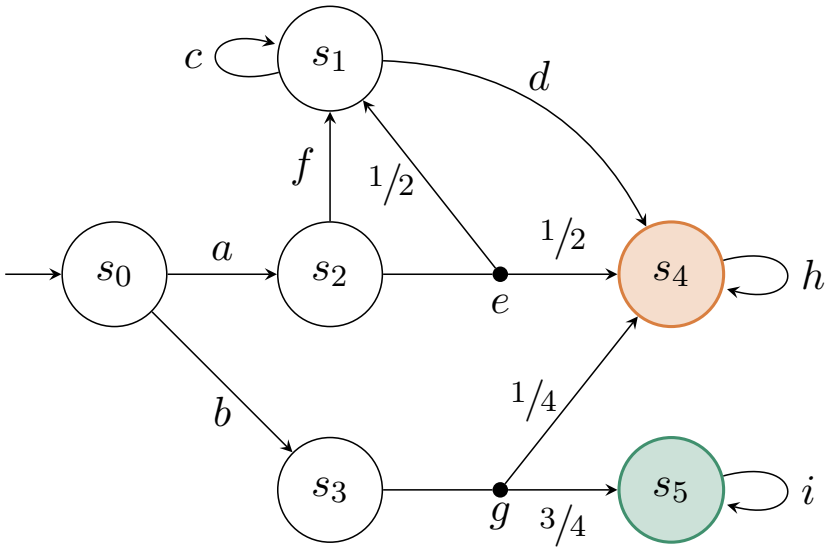

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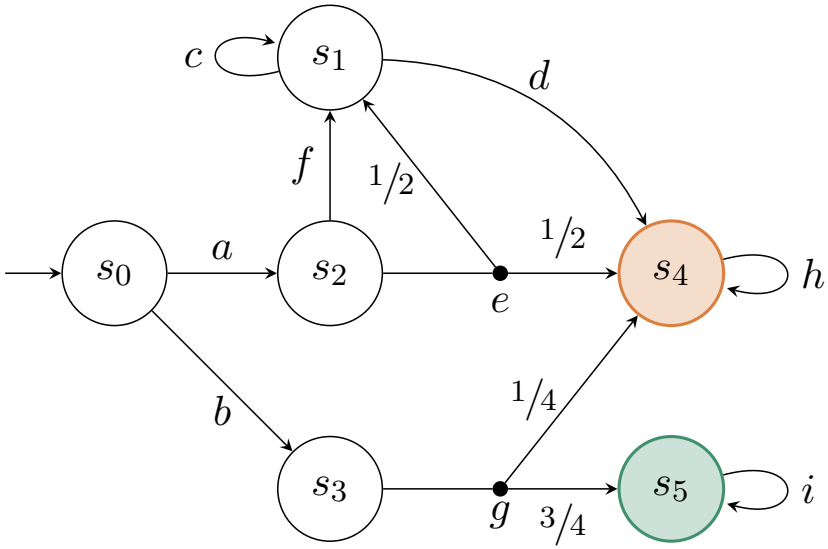

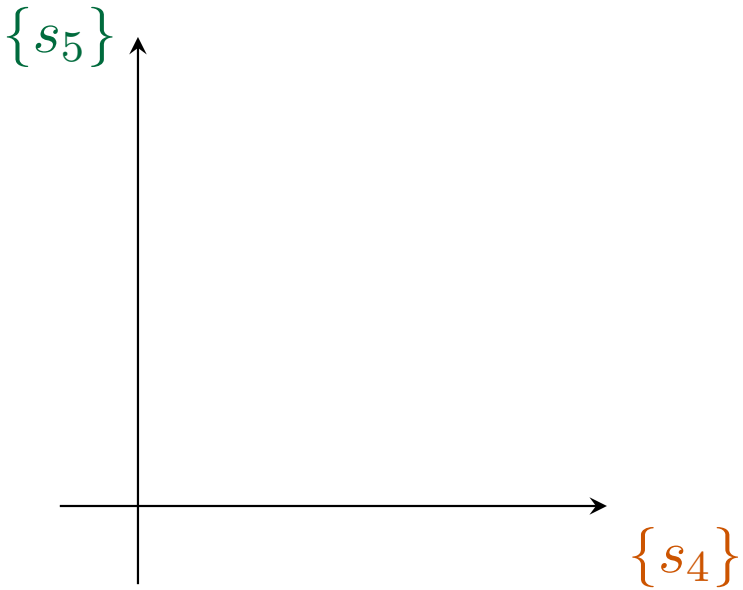

Multi-reachability objectives are vectors of sets of states that we want to **reach**.

A threshold vector $\boldsymbol{\alpha} \in [0,1]^n$ is **achievable** if there exists a strategy σ where $\mathbb{P}_s^\sigma(\Diamond T_i) \geq \alpha_i$ for all $1 \leq i \leq n$.

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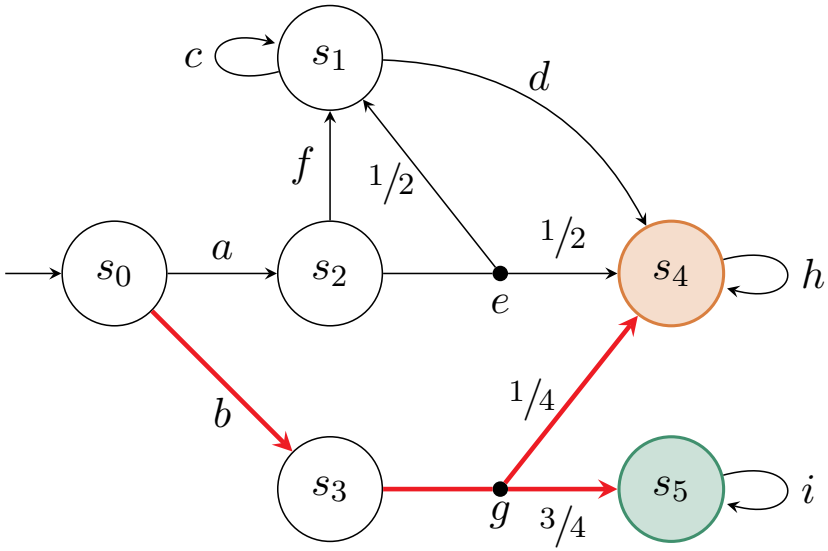

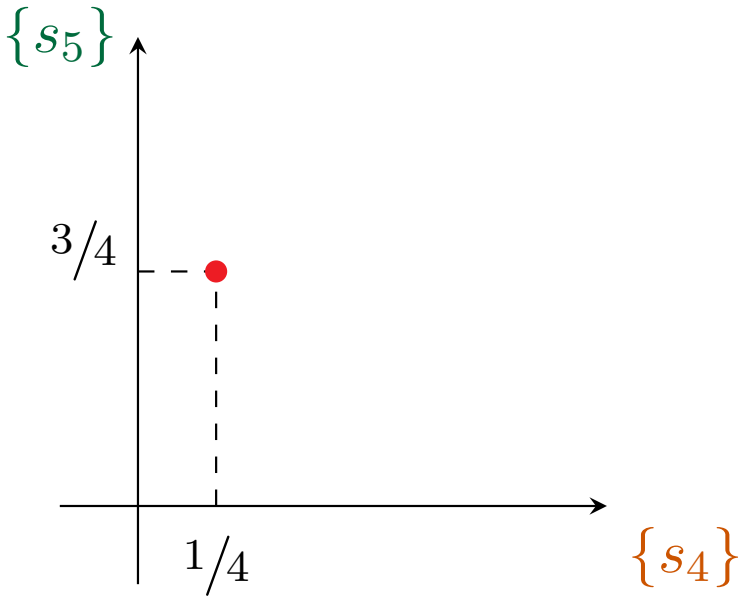

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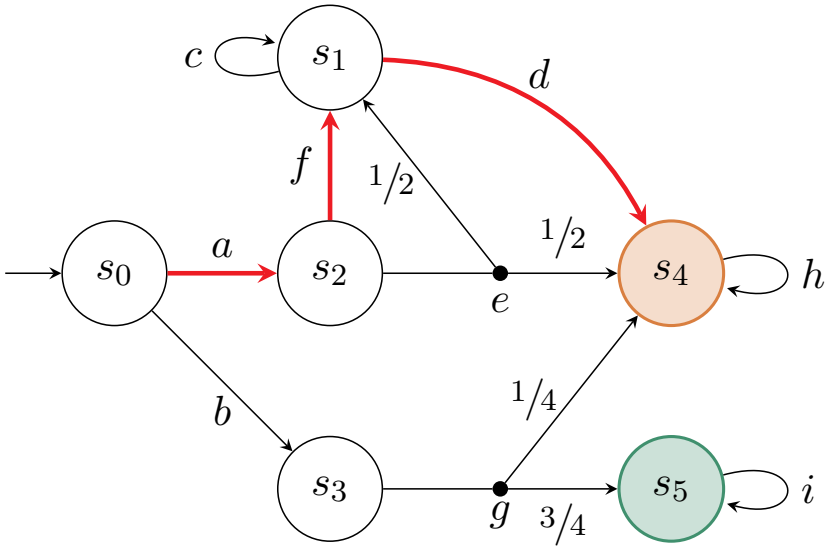

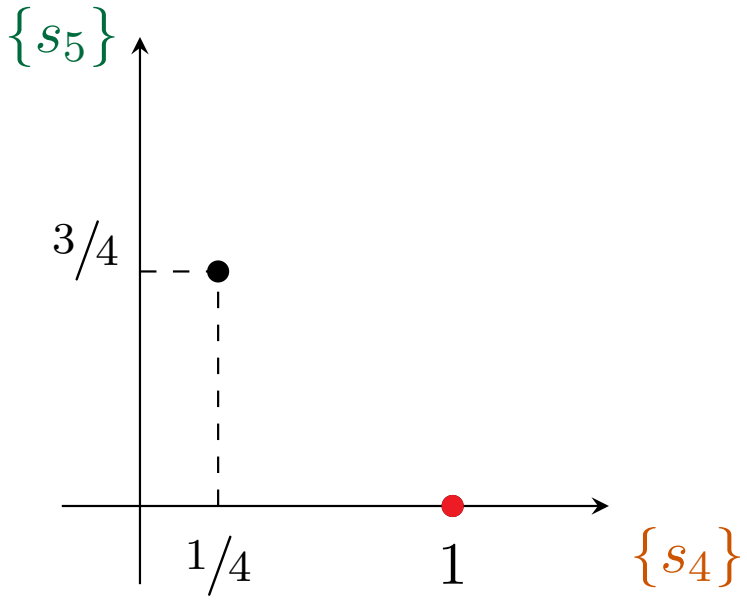

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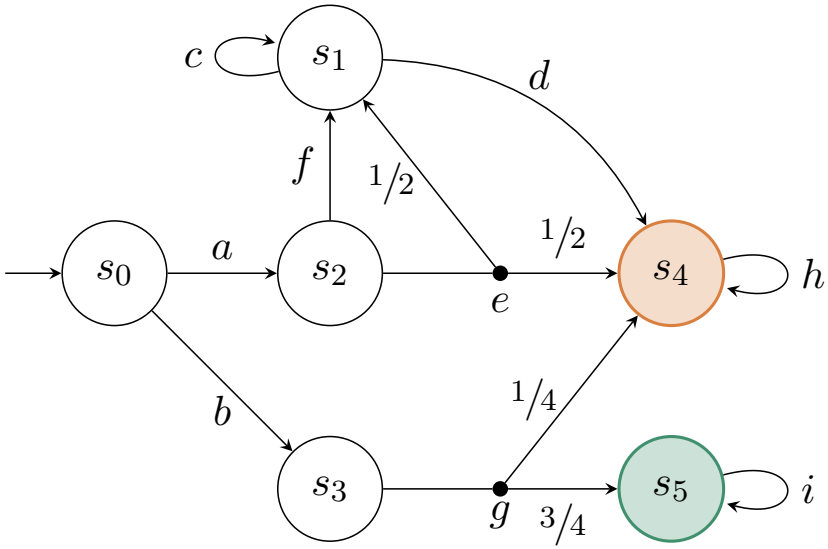

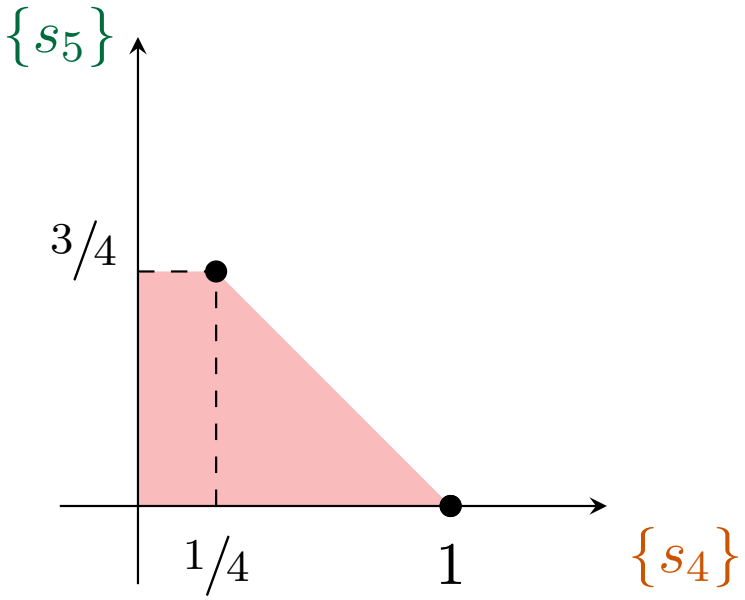

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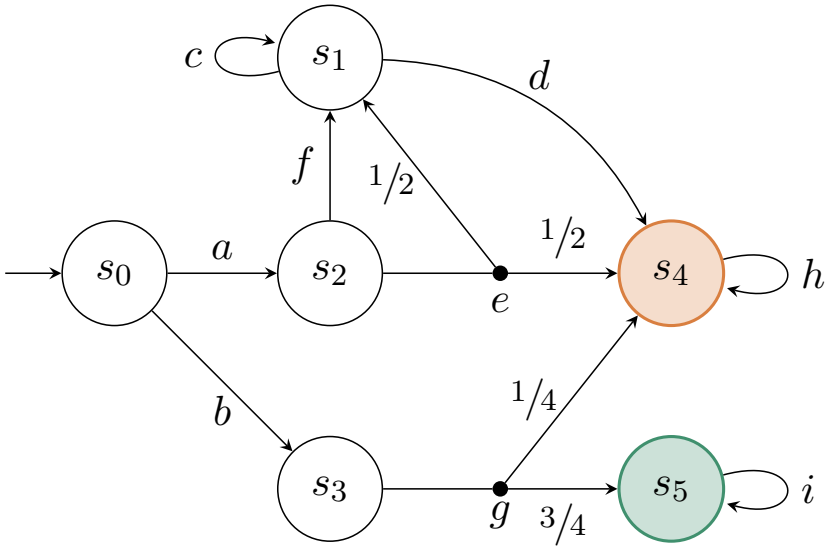

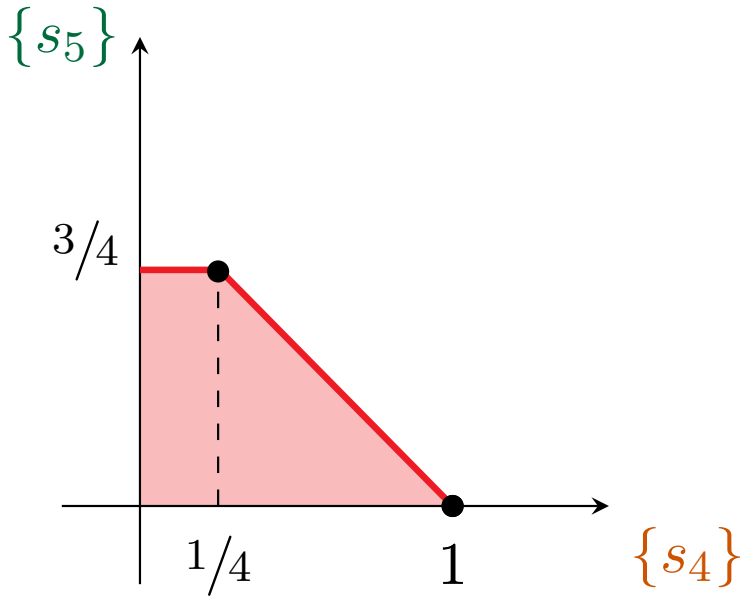

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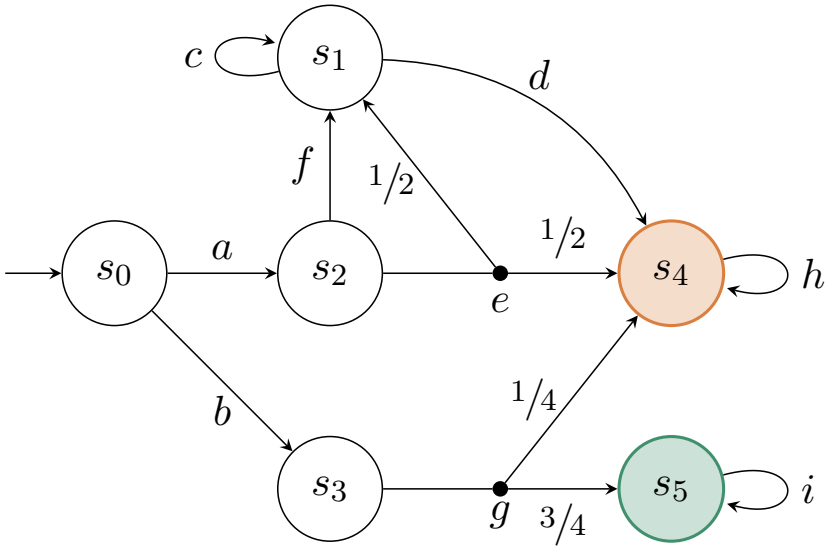

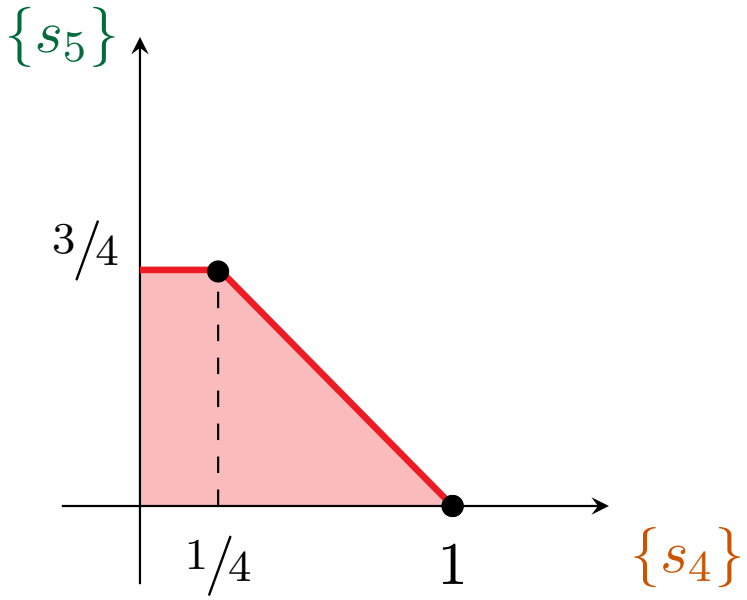

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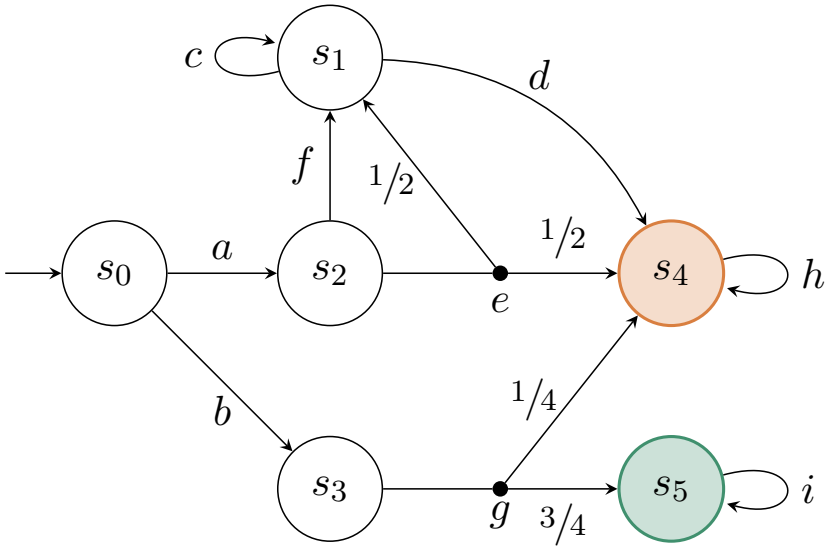

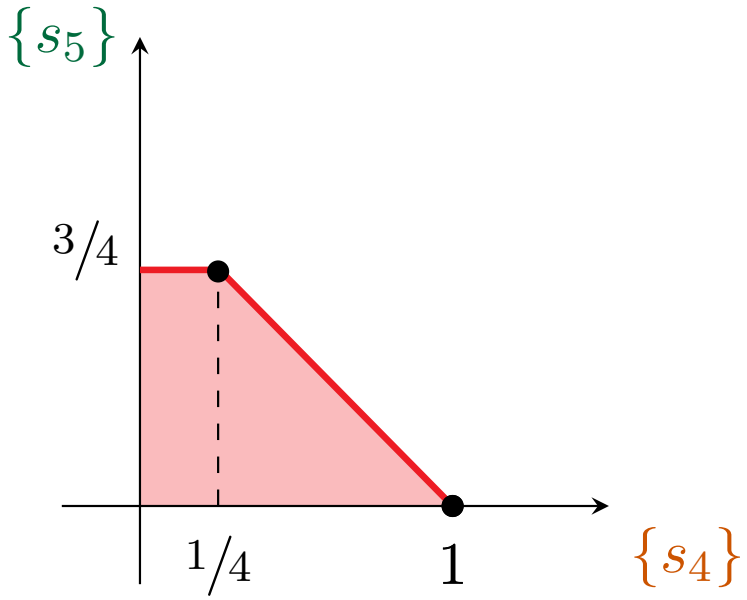

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We want to compute Pareto frontiers $\rightsquigarrow$ Develop **efficient** methods in practice.

Game-based abstraction

We extend the work of [KKNP10][1] from one to multiple dimensions.

Goal. Abstract our model by **merging** states together.

$\rightsquigarrow$ Approximate the Pareto frontier through a smaller model.

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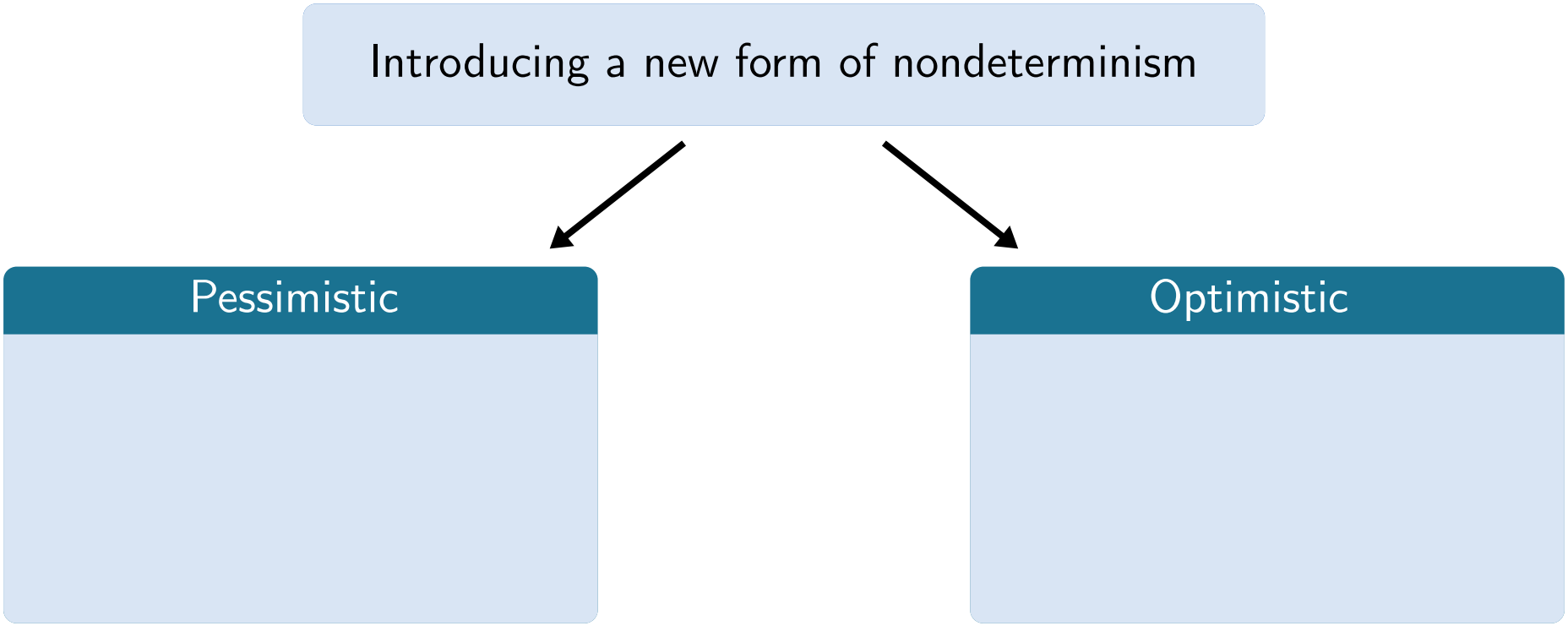

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Pessimistic

- Players are antagonistic (SG)
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Optimistic

- Players are cooperating (MDP)
- Upper approximation of the Pareto frontier

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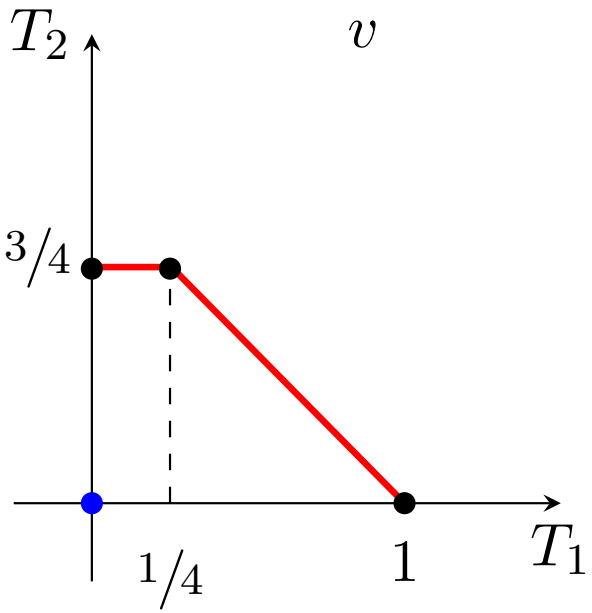
Refining the abstraction

The approximations provide a **quantitative** evaluation of the abstraction's quality and indicate on how to **refine** it.

Example.



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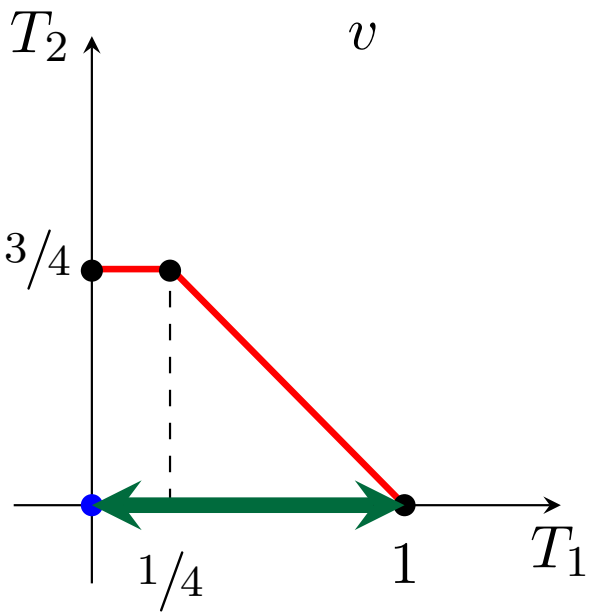

- Consider an abstract state v
- Check the distance between the approximations
- If the distance is too big $\rightsquigarrow$ split the state v

But **how** do we split ?

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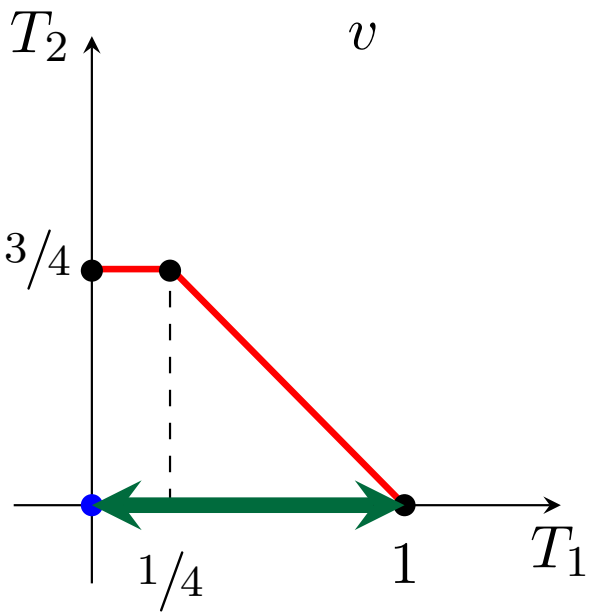

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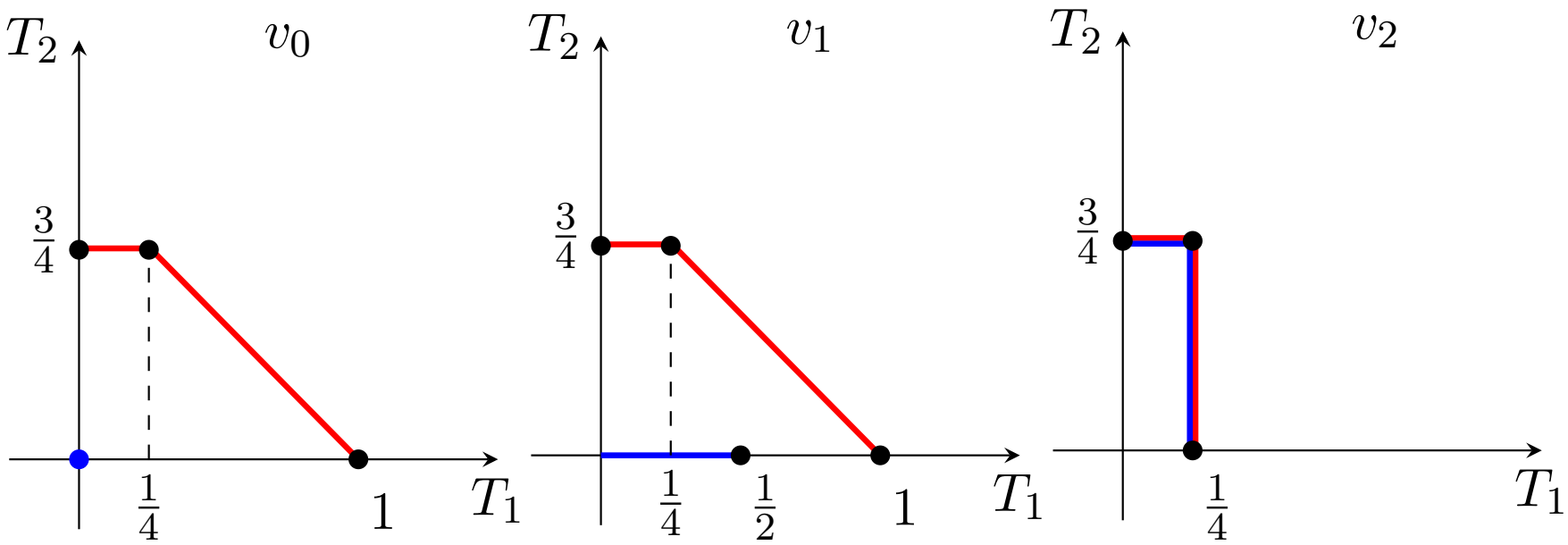
We look at the approximations of the concrete states **contained** in v.

Splitting an abstract state

Example. For $\boldsymbol{d} = (1, 0)$:



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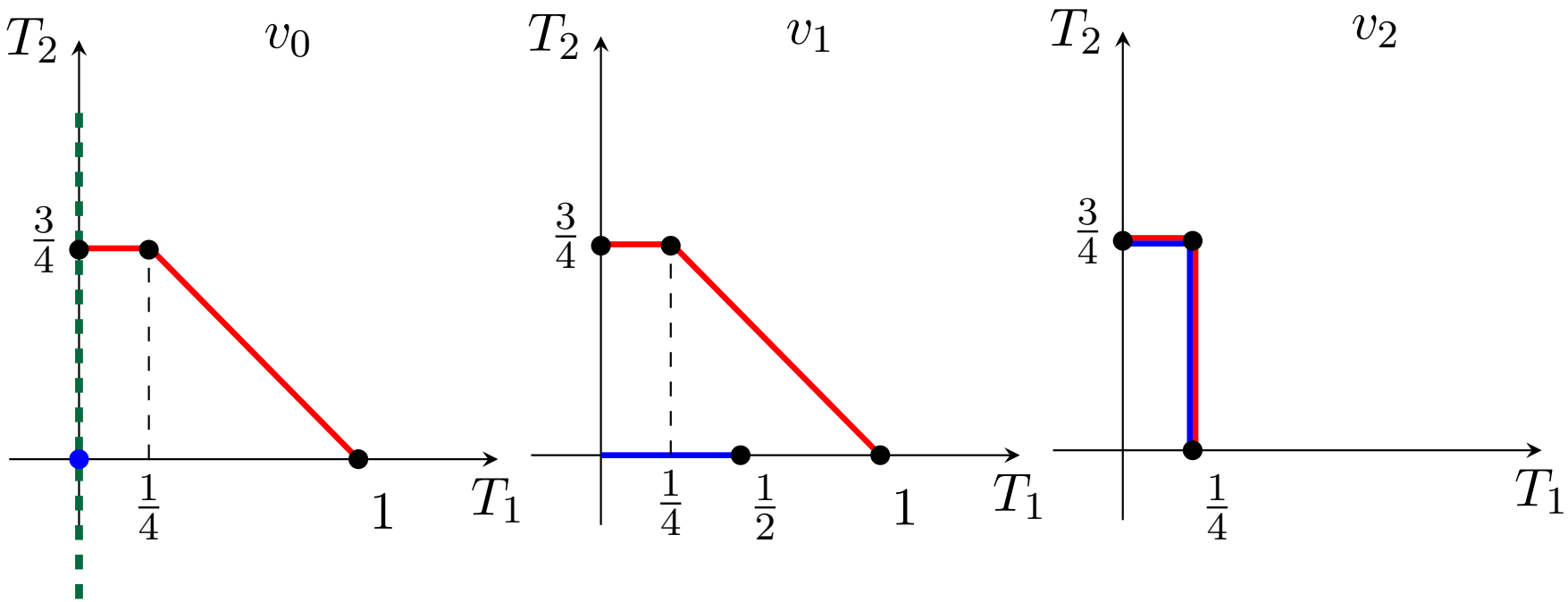

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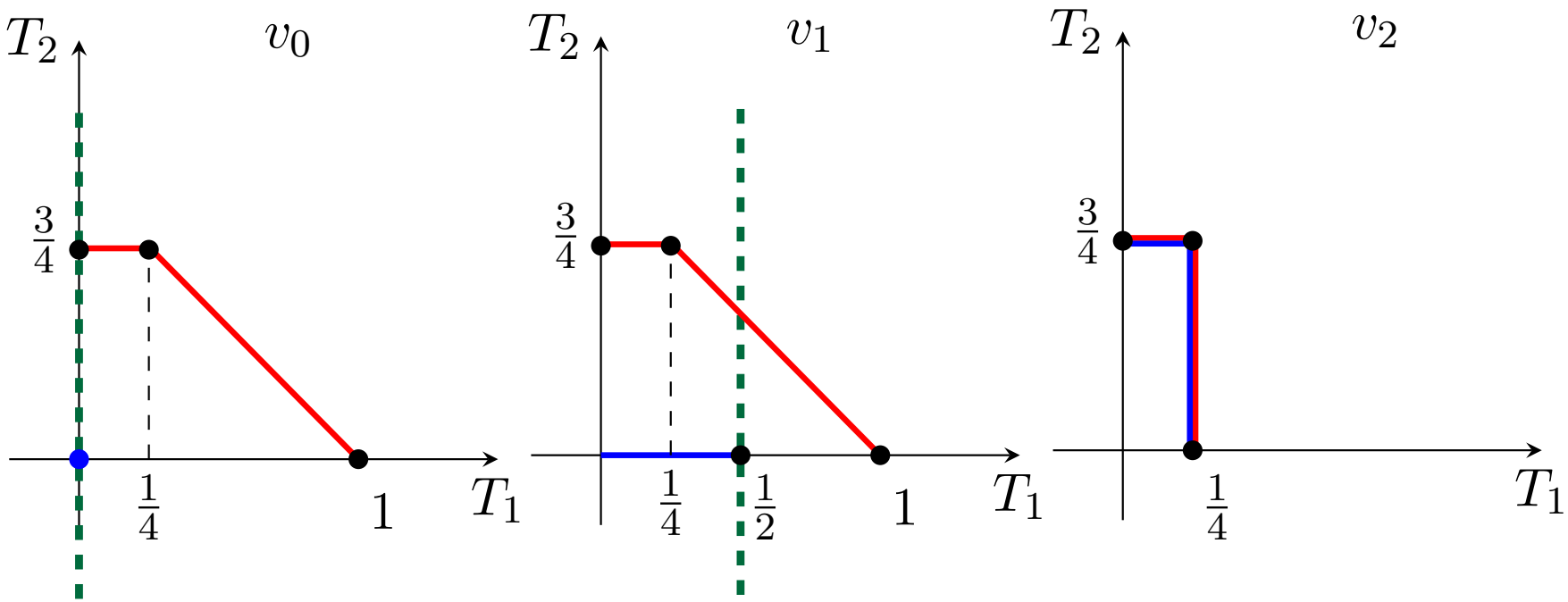

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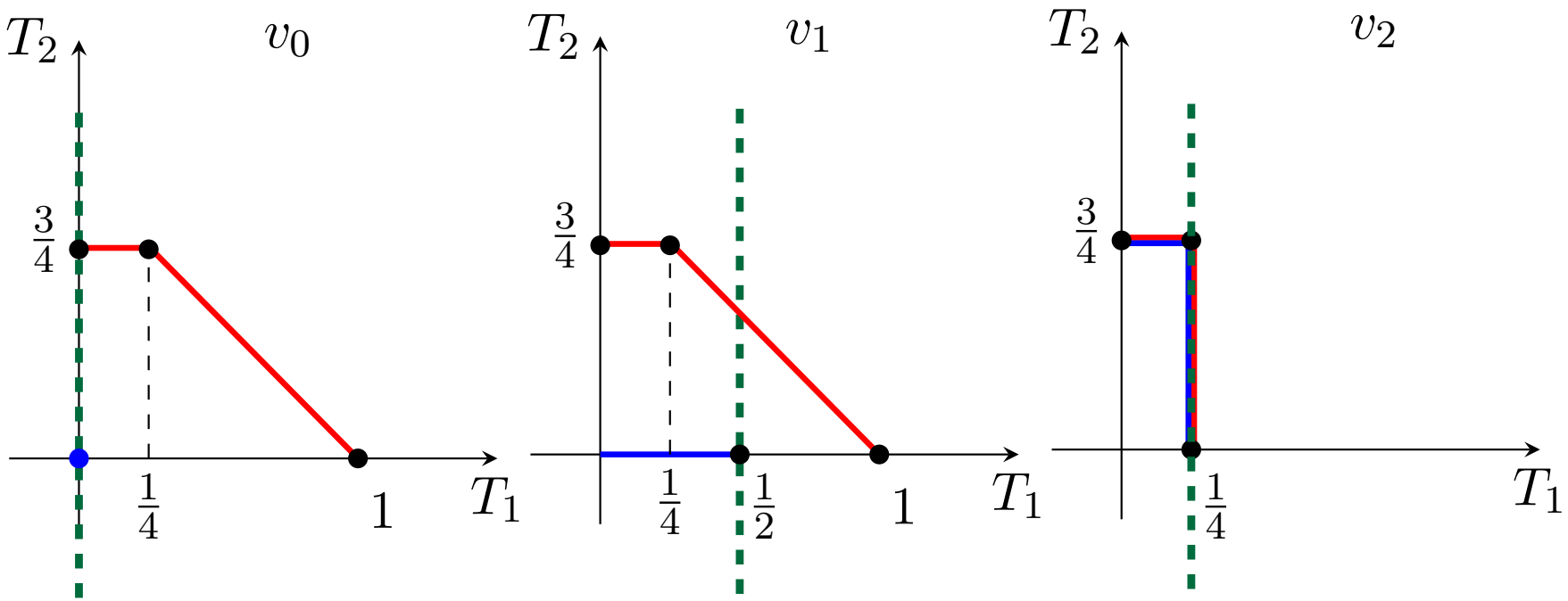

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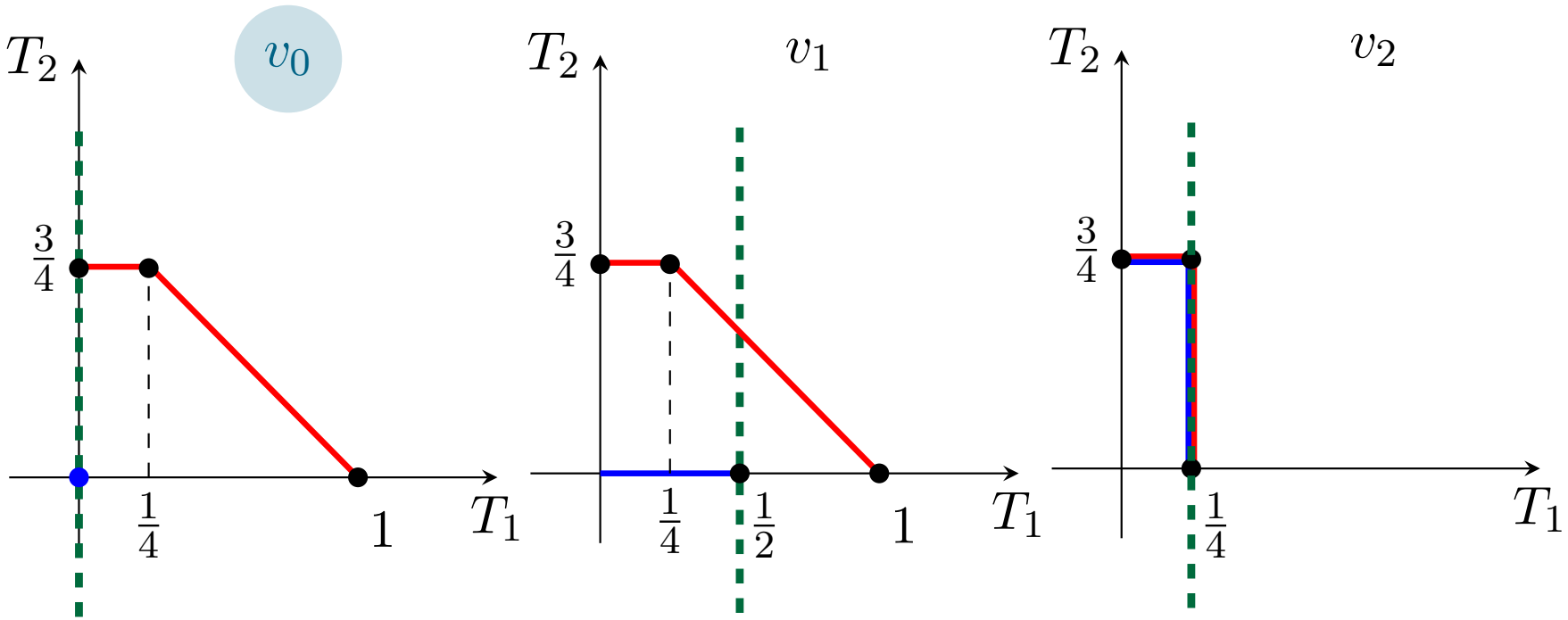

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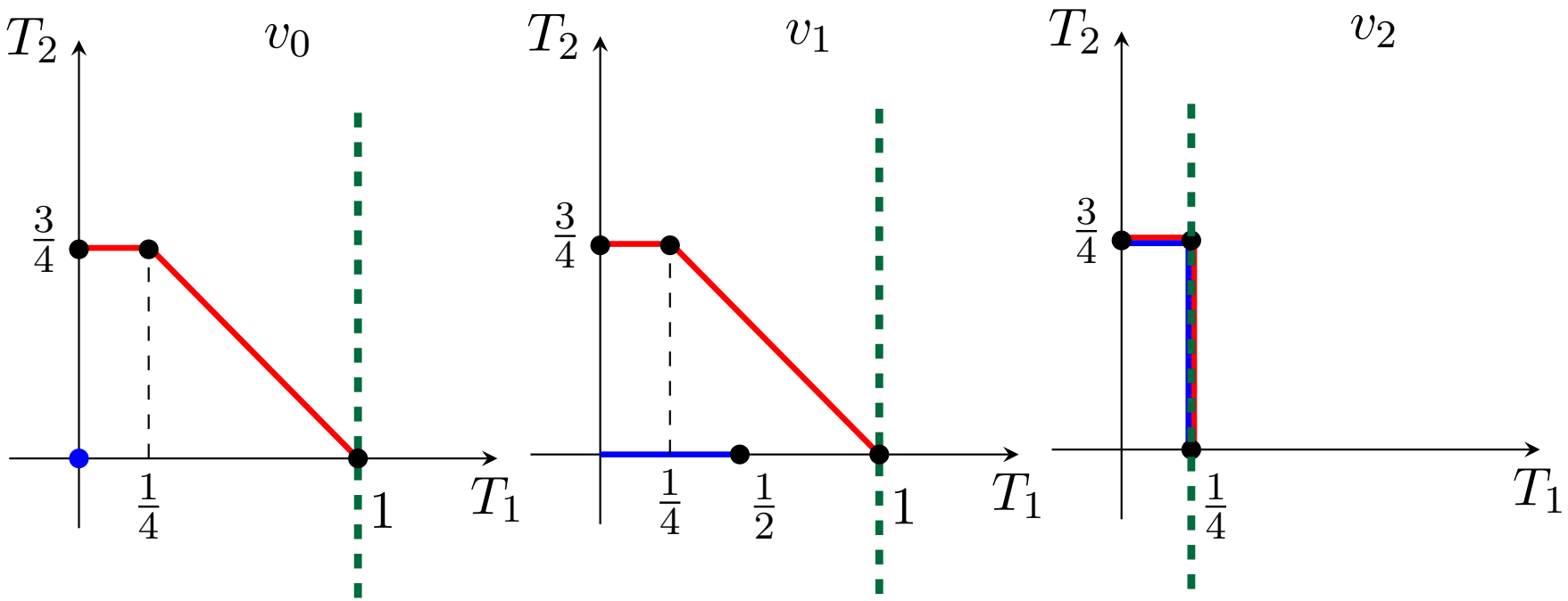

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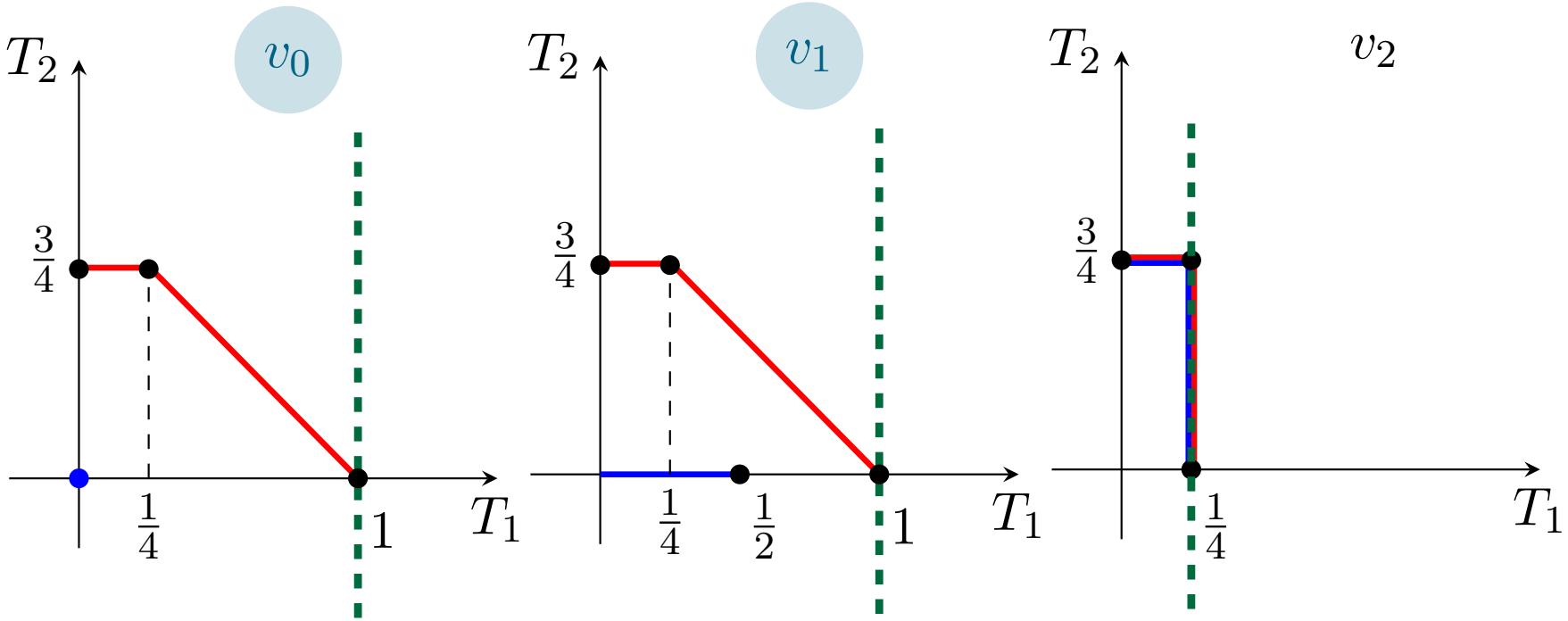

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Thank you for your attention!

Bibliography


Kattenbelt, Mark et al. “A game-based abstraction-refinement framework for Markov decision processes”. In: *Formal Methods Syst. Des.* 36.3 (2010), pp. 246–280. DOI: 10.1007/s10703-010-0097-6. URL: https://doi.org/10.1007/s10703-010-0097-6.